

the optical effect of an array of dislocations are (1) derivation of the stress field of an infinite array of regularly spaced dislocations, (2) determination of the optical effects of this stress field, and (3) estimation of the applicability of this result to real arrays by a comparison of infinite and finite arrays.

The stress field of an infinite array of uniformly spaced dislocations consists of local stresses in the vicinity of each dislocation and a long-range stress corresponding to the average strain required by all the dislocations. The local stresses die out exponentially with distance away from the array and are insignificant at distances greater than the dislocation spacing. The derivation of the local stress field of dislocations and uniform arrays of dislocations involves complex calculations for crystals of low symmetry and these have only been made for a few cases, including hexagonal crystals (Chou, 1962) and α -quartz (Chou, 1963). The calculation of the long-range stress field, on the other hand, involves relatively simple elastic calculations. Since the optical effects are observed at distances much greater than the dislocation spacing in our arrays, we give below the calculations for the long-range stresses and strains.

The case with which we are concerned here is an array of pure edge dislocations with spacing (h), Burgers vector (b) parallel to the a -axis (x_1) and dislocation lines parallel to the base and normal to a (that is, parallel to x_2). We consider only those stresses and strains which have opposite signs on opposite sides of the plane of the array, since only these are the direct reflection of the presence of the array. A normal strain ϵ_{11} of magnitude $b/2h$ is required parallel to the Burgers vector, compressive on the side of the array containing the extra half-planes, and tensile on the other side. The normal strain parallel to the dislocation line (ϵ_{22}) is zero. In order that the dislocations of the array be of pure edge type, it is necessary that $\epsilon_{12} = 0$. If the normal and shear stress components σ_{i3} across planes parallel to the array had opposite signs on opposite sides of the array, gross

equilibrium could not obtain, hence these stress components must vanish. These stress and strain components determine the stress and strain tensors:⁶

$$\sigma_{11} = \frac{b}{2h} \left(c_{11} - \frac{c_{13}^2}{c_{33}} - \frac{c_{14}^2}{c_{44}} \right) \\ = 4.01 \times 10^{11} \frac{b}{h} \text{ dynes/cm}^2,$$

$$\sigma_{22} = \frac{b}{2h} \left(c_{12} - \frac{c_{13}^2}{c_{33}} + \frac{c_{14}^2}{c_{44}} \right) \\ = .53 \times 10^{11} \frac{b}{h} \text{ dynes/cm}^2,$$

$$\sigma_{33} = \sigma_{23} = \sigma_{13} = \sigma_{12} = 0$$

$$\epsilon_{11} = .500 \frac{b}{h},$$

$$\epsilon_{33} = -.070 \frac{b}{h},$$

$$\epsilon_{23} = -.150 \frac{b}{h},$$

$$\epsilon_{22} = \epsilon_{13} = \epsilon_{12} = 0.$$

The effect of this stress field on the indices of refraction are (Pöckels, 1906, p. 484)

$$\Delta n_{11} = -\frac{\omega^3}{2} (\pi_{11}\sigma_{11} + \pi_{12}\sigma_{22}) = .107 \frac{b}{h},$$

$$\Delta n_{22} = -\frac{\omega^3}{2} (\pi_{12}\sigma_{11} + \pi_{11}\sigma_{22}) = .196 \frac{b}{h},$$

$$\Delta n_{33} = -\frac{\omega^3}{2} [\pi_{31}(\sigma_{11} + \sigma_{22})] = .236 \frac{b}{h}.$$

When the c - and a -axes are in the plane of the thin section, the difference in birefringence on each side of an array of this type is

$$\Delta \text{Biref.} = \Delta n_{33} - \Delta n_{11} = .129 \frac{b}{h}.$$

Compression increases both the indices and birefringence; tension reduces both. It has been noted that lamellae show greatest con-

⁶ Values of the stiffness coefficients (c_{ij}) are from Huntington (1958).

trast when viewed in light vibrating parallel to ϵ and least in light parallel to ω . It was found that this difference in contrast was about three times as great parallel to ϵ as parallel to ω , consistent with the values (Δn_{33} and Δn_{11} , respectively) calculated above.

The stress (σ_{11}), dislocation spacing (h), and number of dislocations per centimeter of lamella (N) corresponding to the observed change of birefringence along average lamellae (assuming $b = 4.91 \text{ \AA}$) are

$$(\Delta n_{33} - \Delta n_{11}) = .0005; \quad \sigma_{11} = 1.5 kb; \\ h = 250 \text{ } \mu\text{m}; \quad b = 1300 \text{ \AA}; \quad N = 8 \times 10^4/\text{cm}.$$

Electron microscope photographs of etched lamellae reveal lines of closely spaced etch pits parallel to the lamellae (pl. 4, A). The average number of etch pits per centimeter in this photograph is $5 \times 10^4/\text{cm}$ and the closest-spaced are $13 \times 10^4/\text{cm}$. It has not yet been possible to obtain both optical and etch-pit measurements on a single lamella. The separation of lines of etch pits is less than the optical resolution in about one-third of the pairs in plate 4, A, suggesting that some optical lamellae may be compound arrays.

It has been verified optically that lamellae as seen in transmitted light are coincident with lines of etch pits seen microscopically in reflected light. The electron microscope resolves these etch pits and shows the pattern characteristic of lines of etch pits along slip planes in crystals whose slip system is known from other evidence. The approximate correspondence between etch-pit spacings and observed optical effects suggests that etch pits are the sites of individual unit dislocations.

The difference between a finite and an infinite array may be estimated by calculations on continuous arrays in an isotropic medium. On the median plane of such an array the principal stresses are

$$\sigma_{xx} = \frac{\mu}{\pi(1-\nu)} \frac{b}{h} \left(2 \cot^{-1} a - \frac{a}{1+a^2} \right),$$

$$\sigma_{yy} = \frac{\mu}{\pi(1-\nu)} \frac{b}{h} \left(\frac{a}{1+a^2} \right),$$

where $a = 2y/l$ and l is the length of the array in the direction of the Burgers vector (x); y is the distance at which the effect is being observed; and the array is infinite in the direction of the dislocation lines (z). When a is zero, the array is infinite in the x direction. The ratio of the largest stress (σ_{xx}) in finite arrays to that for infinite arrays of the same kind is as follows:

l/y	$\sigma_{xx}/\sigma_{xx\infty}$
∞	1.0
100.....	0.98
40.....	0.95
20.....	0.91
10.....	0.81

Hence, when birefringence is observed at a distance from the lamellae less than $\frac{1}{10}$ of their length, as is usually the case, the value for an infinite array should be a good approximation.

The quartz becomes optically biaxial, and, as a result of the ϵ_{23} strain component, there is a rotation of the principal axes of the indicatrix about the x_1 -axis (a). The magnitude of this rotation is approximately $\frac{1}{2}^\circ$, in opposite senses on the two sides of the array. Most of our thin sections were prepared parallel to the x_1x_3 -plane ($a-c$), and no changes of extinction position were observed in these sections. In the only sample sectioned perpendicular to x_1 (a), rotations of less than 1° were observed close to the lamellae.

Arrays of basal edge dislocations are thus observed to have all the optical properties observed in basal lamellae. Other types of dislocations may be ruled out. "Walls" of dislocations are excluded, since the stress decreases exponentially with distance and would not produce observable changes of indices or birefringence in quartz. Basal or prismatic arrays of screw dislocations would produce no change in refractive index parallel to x_1 , x_2 , or x_3 since the normal stresses parallel to these axes are zero. In the case of a basal array of screw dislocations parallel to the a -axis, the quartz would become biaxial and the principal axes of the indicatrix would be inclined to the reference